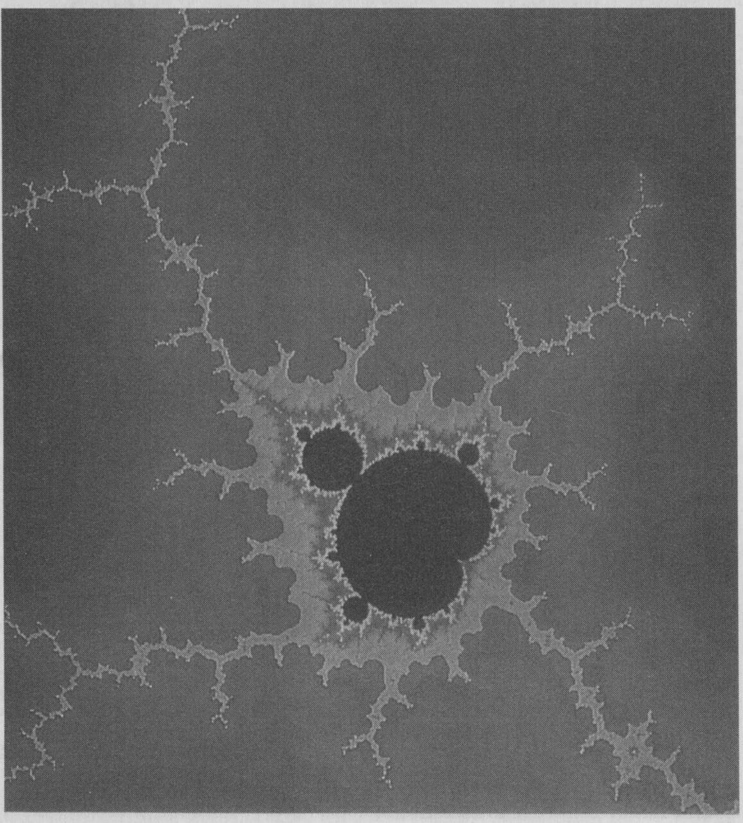
FEB 1990

Cash from chaos

Nick Brooks voyages into the strange and beautiful universe of the Mandelbrot Set – the mathematical expression of a revolutionary way of viewing nature



At the heart of chaos theory lies the Mandelbrot Set – an immensely complex mathematical shape which expresses a whole new way of understanding natural phenomena

The Mandelbrot Set lies like a black hole, a vortex of infinite complexity at the centre of an imaginary two-dimensional sea of numbers called the complex plane. From a distance, the Set could be a psychoanalyst's inkblot, surrounded by multicoloured waves lapping against its shores.

It seems strange that this biological-looking shape should lie at the heart of a new science, and a new attitude to the way we understand and predict natural phenomena – chaos theory. But this is the one and only Mandelbrot Set – the most complex mathematical object ever seen.

There is no mysterious alchemy at work here. Each point on this complex plane has been tested against a terse algorithm many tens, hundreds or thousands of times. Inside the Set, the points which have been captured and are unable to escape after repeated testing are displayed as black. Outside the Set are the points which have escaped before testing was completed, having near infinite values, and they are displayed as coloured.

The Set is named after Benoit B Mandelbrot, a mathematician and research fellow from IBM's Research Centre at Yorktown Heights, New York. Dating from 1979, the Set has risen to become something of an emblem of chaos and the brightest star in modern mathematics. It was Mandelbrot's explorations into the world of geometry which led to this and other discoveries, which he named, collectively, fractal geometry.

Fractal geometry

> "Why is geometry often described as "cold" and "dry"? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not in spheres, mountains are not cones, coastlines are not circles and bark is not smooth, nor does lightning travel in a straight line. More generally I claim that many patterns of nature are so irregular and fragmented that, compared with Euclid – a term used in this work to denote all standard Geometry – Nature exhibits not simply a higher degree, but an altogether different level of complexity. The number of distinct levels of natural patterns being for all practical purposes infinite".
>
> *Benoit B Mandelbrot – The Fractal Geometry of Nature*

How long is the coastline of Britain, and is it even possible to accurately measure it? Think about it – if you were to examine the southern coast of Cornwall from a satellite, and then examine a part of the same coastline on your knees, would one seem less complex than the other? No. The pattern of bays and peninsulas visible from space would be present in a microscopic scale as you leant over the side of a rock pool.

If you were then to examine a square millimetre of the edge of the rock pool, you would again see a complexity of pattern and form equalled by the view from space. To all intents and purposes, this macro-to-micro scale of complexity of pattern and form is infinite. The closer you examine a natural object, the more detail is visible. It is the existence of this seemingly unmeasurable and recursive self-similarity in nature that we see at the boundary or shore of the Mandelbrot Set.

Micro-worlds

With the aid of a simple program, we convert our PC into a microscope and start our voyage into the complex micro-world of the Mandelbrot Set. Our voyage takes us closer to the boundary separating the Set from the sea of colours to reveal an intricate and minute coastline of instability. Shapes looking like seahorses' tails and islands of molecules appear in microscopic detail. Shooting further into the crook of one of these tails, we see new patterns like solar flares and frost crystals loom into being from seemingly chaotic areas of colour. We travel further, aiming towards an area of black embedded like a dark jewel in a spiral of colour. Approaching it, we see the area is a sibling set of the Mandelbrot Set, many thousands of times smaller, but still familiar. It is equally as beautiful and complex, but never identical, with its own coastline of instability and chaos.

This is an infinite world. There is no end to the recursion of complexity of pattern, form and self-similarity on these alien coastlines. We will never find a repetition or a straight line, which – if it existed – would mark the end of our fractal journey.

Programming

The Mandelbrot program needs just a few essential pieces. The central engine of the program is a remarkably simple algorithm:

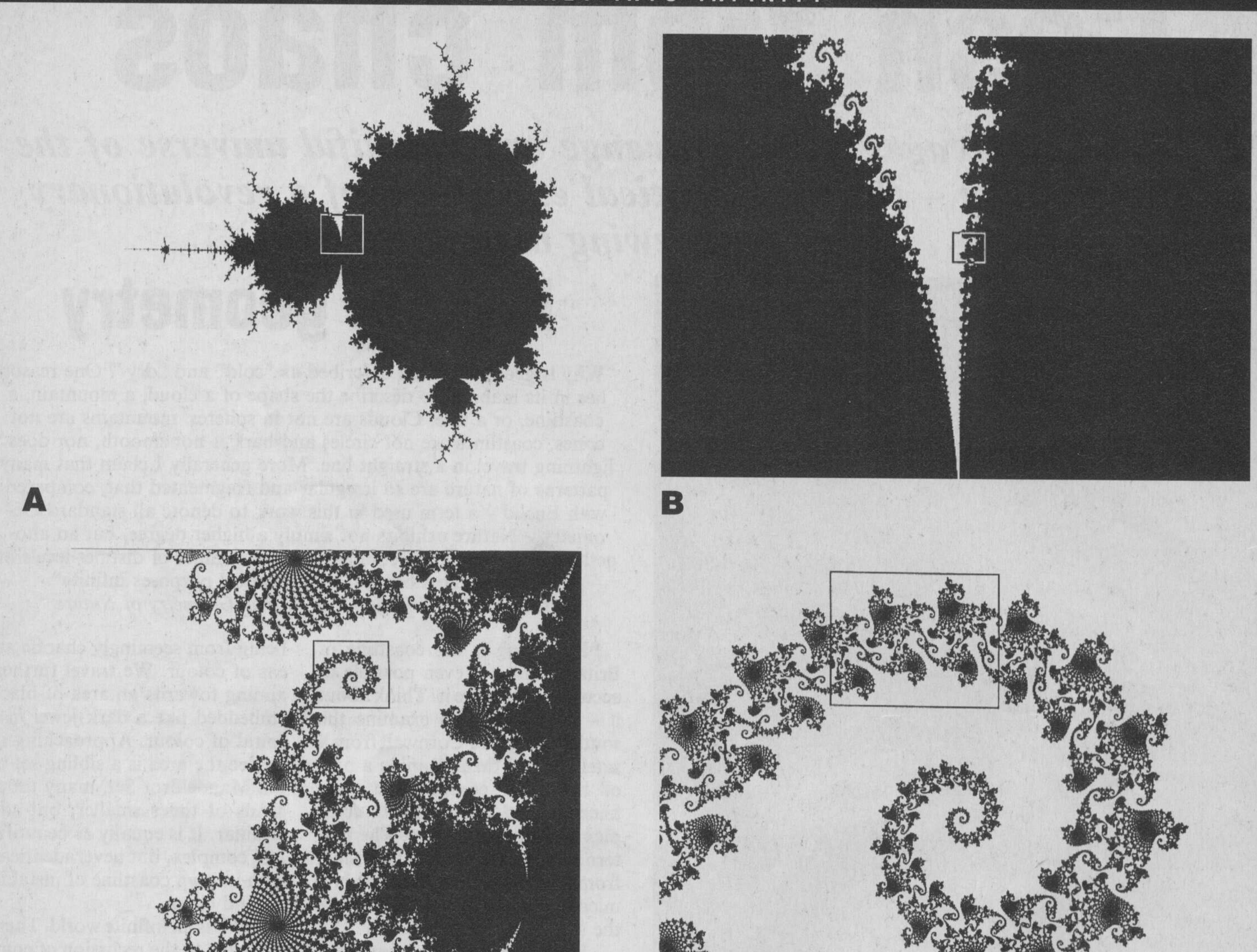
$$z \rightarrow z^2 + c$$

where z is a complex number that is allowed to vary and c is a fixed complex number. If, like me, you hold about as much love for algebra as you hold for the Community Charge, don't get too uncomfortable – this is easy! The algorithm is a simple loop which takes a value for z, then squares it, adds c and then passes the result back to z, to give a new starting value to be looped again and again – iterated, in other words.

Complex numbers

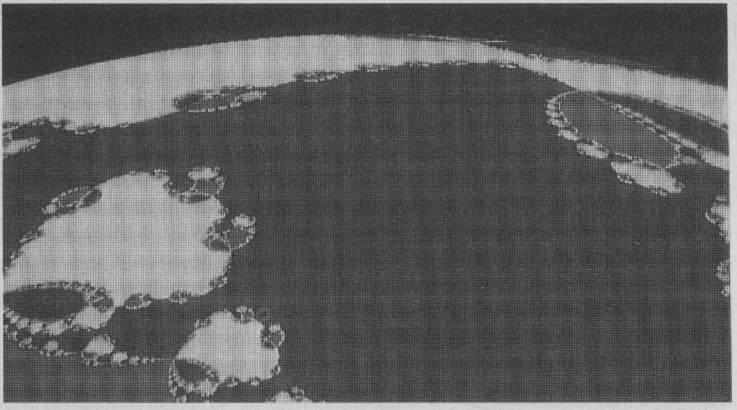
At this point we should take a quick look at what is meant by

JOURNEY INTO INFINITY



Just as the patterns visible in a coastline seen from space are reproduced in a microscopic portion of the edge of a rock pool, so the Mandelbrot Set reproduces itself infinitely. Figures A to F show progressive magnifications of portions of the Set until, in the middle of figure F, we can see a tiny sibling set of the Mandelbrot Set itself emerging

All illustrations from *The Beauty* [illegible] *Fractals* by Peitgen and Richter, published by Springer-Verlag



complex numbers. Created and agonized over by 19th Century mathematicians, complex numbers were created to combine real numbers with an imaginary value of sqrt.–1. Interestingly, they are a very good example of how pure and abstract 19th Century mathematical theory was, and have found remarkable applications in today's electronic technology.

A complex number is made of two parts which, for purely historical reasons, are called real and imaginary. For example, 2+3i is a complex number with real part 2 and imaginary part 3i. The i next to the 3 has a value of sqrt.–1, and also shows which part of the number is imaginary. The modern view of complex numbers is that they are most simply defined as points or co-ordinates in a plane – the complex plane. Thus 2+3i is the address for a point 2 east and 3 north on the complex plane.

Adding complex numbers (addition and multiplication are used in the algorithm) is simple. Just add the real parts together and the imaginary parts together:

2+4i
+ 9–2i
= 11+2i

Multiplication is as simple, remembering that i squared = –1:

2+3i
x 2+3i
6i+$9i^2$
4+6i
4+12i+$9i^2$
4+12i–9
= –5+12i

When complex numbers are applied to z –> z^2 + c iteratively, strange things begin to happen to particular values of c. For example, here is what happens when c is 1+i:

First iteration: 1+3i
Second iteration: –7+7i
Third iteration: 1+97i

Note that the real and imaginary parts may grow, shrink or change sign after each iteration. If this process continues, the resulting values for z will get progressively larger, and very shortly will scream off into infinity.

Border skirmishes

Obviously, when programming the Set, we must have some means of breaking out of the z –> z^2 + c loop. Otherwise, after just a few iterations, we will exceed the capacity of any computer with near infinite values for z. So we adopt two methods for breaking out of the loop, or bounding.

1) The program must watch its running total. If the value for z exceeds 2, the theory of complex number iteration guarantees us that further iteration will drive z off into infinity – and therefore z can be assumed to be outside the Set and the program will move to the next point to be tested.

2) We can simply limit the amount of iterations the loop can make, thereby saying that any point on the complex plane which is still <= to 2 after this iteration limit can be assumed to be a member of the Set, and again the program will move on.

After each point on the complex plane has been tested according to these simple rules, the program displays the result graphically. All points inside the Set are coloured black. Those points outside the Set are assigned colours according to how soon they break out of the loop. For example, those points breaking out after three iterations might be displayed as green, after four red, after five blue, and so on. A zoom lens for the Mandelbrot is the next essential thing any would-be fractal traveller should create for the program, without which the true beauty and intricacy of the Set will remain hidden. This a relatively simple task. Generate two cursors which read values for the ranges and then input these as the parameters for a new drawing – creating a fully interactive micro-viewport to the Set. The power of this zoom lens will depend to some degree on the effective size of numbers that your PC can manipulate. An Amiga will produce magnifications in the order of x800,000. If the Amiga was a real microscope, its power would equal that of an electron microscope.

Other useful additions to your program could include a screen-save procedure to save your fractal snapshots for easy retrieval without lengthy recalculation and redrawing. Or perhaps giving the colour values an additional height value which, when combined with a perspective viewing procedure, will create a contoured rock relief of the Mandelbrot Set.

But the most compelling and aesthetically-pleasing images are created by careful manipulation of the colour palette. Examples of these intricate images can be seen in the horde of articles and books published on fractals. ▶88

THE MANDELBROT SET

(In Borland Turbo C)

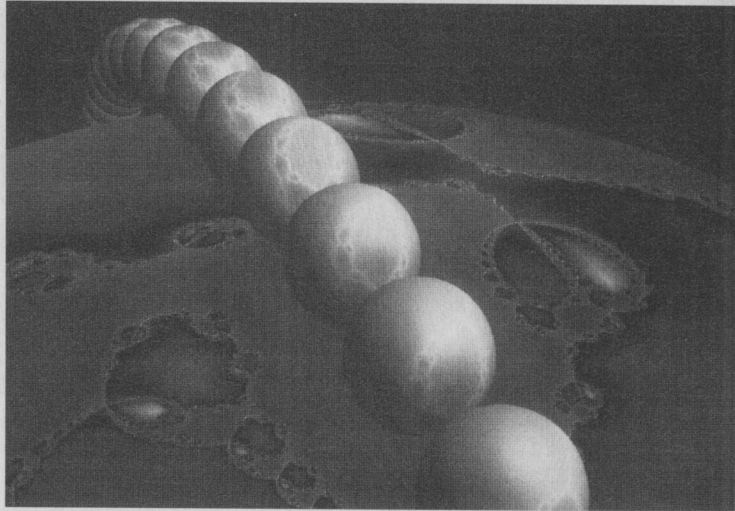
```
#include <graphics.h>
#include <math.h>
#include <conio.h>
#include <alloc.h>
#include <stdio.h>
#include <dos.h>
 int graphdriver=DETECT, graphmode, iters=32, i, j, k, it;
    float scale = 1.0, xrange = –2.5, yrange = –1.5;
    float cr, ci, zr, zi, zm, z1, z2, zr2, zi2;
    int a[4]={0, 1, 0, 1}, b[4]={0, 1, 1, 0};
main()
{
   initgraph( &graphdriver, &graphmode, “”); /*initialis-
esscreen*/
    for(i=0; i<=3; i++)
    {
        for(j=a[i]; j<=getmaxy()/scale; j+=2)
        {
            for(k=b[i]; k<=getmaxx()/scale; k+=2)
            {
                cr=xrange+k*4/(getmaxx()/scale);
                ci=yrange+j*3/(getmaxy()/scale);
                zm=zr=zi=zr2=zi2=0.0;

                  /* Mandelbrot engine */

                for(it=–1; (it<iters) && (zm<4); it++)
                {
                    z1=zr2–zi2+cr;
                    z2=2*zr*zi+ci;
                    zr=z1;
                    zi=z2;
                    zr2=zr*zr;
                    zi2=zi*zi;
                    zm=zr2+zi2;
                }
                if (it==iters)

                    /* drawing commands */

                putpixel(k, j, (int)(fmod(it+2, 16)));
                if (kbhit()) exit();

                }
            }
        }
}
.s
off
```

RECOMMENDED READING

The Science of Fractal Images – Peitgen and Saupe (Springer-Verlag).
The Beauty of Fractals – Peitgen and Richter (Springer-Verlag).
Chaos – James Gleick (Cardinal).
The Fractal Geometry of Nature – B Mandelbrot (Freeman).

Disorderly behaviour

> "Art is a lie that lets us recognize the truth."
> Pablo Picasso



Fractals are everywhere. They are all around us. In the texture of bark, in the shape of clouds and in the flickering of a flame. To a non-scientist, it may seem odd that such familiar things as these have become the focus for such intense research. But familiarity is not enough to ensure that scientists have the tool for an adequate understanding. In fact, many scientists and thinkers have become aware of the inadequacies of the established wisdom in areas as seemingly fundamental as natural geometry. Fractals answer many of these questions which, all too often, had been shied away from before because they were considered too complex, and therefore unsolvable.

The benefits of chaos theory are now being felt strongly in a complete sweep through many disciplines and sciences. Fractals will soon be used to compress graphics data into small enough packages to pass along telephone lines, making possible new forms of visual telecommunication such as the videophone. In other areas of research, such as economics, fluid dynamics, meterology and anthropology, chaotic algorithms are being used to model natural behaviour graphically, with an attention to detail not available before. They can tell us more about how well we can reliably predict, for example, economic trends, as opposed to building simplistic models which pay little or no attention to the chaotic behaviour present in many large and even small scale systems.

In films, fractals are being used to generate special effects, as in *Star Trek*'s Genesis effect – the cosmic rebirth of a planet – which used fractal procedures to create the fire and mountains for the sequence.

It is safe to predict that most graphics software and hardware will soon have fractal procedural primitives for the manipulation of surfaces, enabling users to create natural-looking models of mountains, water, clouds, trees and so on, all in the quest for realism (the Symbolics MacIvory Graphics Workstation already has many of these facilities).

The Mandelbrot Set is the most complex of all these fractals, and really is responsible for giving chaos a face. Just to hear a scientist admiring its frightening complexity and intricate beauty, more in the terms of Blake and Byron, forces us to question whether it is art or science – two things that we are traditionally told are as different as intuition and analysis. The answer is simple. It is both.

Benoit Mandelbrot always takes money to the pub with him

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